

Evolution of QCD Coupling Constant at Finite Temperature in the Background Field Method

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Abstract

The evolution of QCD coupling constant at finite temperature is considered by making use of the finite temperature renormalization group equation up to the one-loop order in the background field method with the Feynman gauge and the imaginary time formalism. The results are compared with the ones obtained in the literature. We point out, in particular, the origin of the discrepancies between different calculations, such as the choice of gauge, the break-down of Lorentz invariance, imaginary versus real time formalism and the applicability of the Ward identities at finite temperature.

1 Introduction

One of the application of field theory at finite temperature [1-4] is to find the behaviour of coupling constant as a function of energy, temperature, and the chemical potential using the renormalization group (RG) equation. The knowledge of coupling constant at finite temperature environment then can be used, for instance, in perturbative calculations for quark-gluon plasma created in ion-ion collisions at high energies, in the evaluation of the grand unification scale in a cosmological context and the shift of the energy levels in a hydrogen plasma.

Recently a great deal of attention has been paid to the study of the behaviour of the coupling constant at finite T [5-19]. The resulting formula for the temperature and scale dependent part of the coupling constant has proved to be very sensitive to the prescription chosen. In this paper we wish to examine this and clarify the origin of the discrepancies between different calculations in the literature. We employ the background field method [20,21], which is based on a manifestly gauge invariant generating functional, and the Feynman gauge. The background field method provides notorious simplifications since we have to calculate only the renormalization constant of the background field gluon propagator. We discuss this method in section 2.1. Since the coupling constant depends not only on energy but also on temperature, we derive a pair of RG equations - one for energy and one for temperature. This is done in section 2.2 and in section 2.3 we derive its solution. In section 2.4 we discuss the Feynman rules at finite temperature in the imaginary time formalism.

In order to get the renormalization constant of the background field gluon propagator we have to calculate the polarization tensor. At finite temperature due to the lack of the Lorentz invariance the structure of the polarization tensor is not equivalent to the zero temperature one. In section 3.1 we give a prescription how to define the renormalization constant at finite temperature. In section 3.2 we calculate the polarization tensor in the one-loop approximation. In section 3.3 calculating the vacuum part of the polarization tensor we reproduce the well known zero temperature formula, i.e. the standard vacuum QCD one-loop result [22,23], which reads as

$$[g^2(\mu)]^{-1} = [g_0^2(\mu_0)]^{-1} [1 + 2g_0^2(\mu_0)K_0/(4\pi)^2] , \quad (1.1)$$

where

$$K_0 = (11N/3 - 2n_F/3) \ln(\mu/\mu_0). \quad (1.2)$$

Here N is for $SU(N)$, n_F is the number of flavours, μ is the energy scale and μ_0 is the reference scale.

We calculate the matter part of the polarization tensor in section 4.1 and, in section 4.2, we derive the temperature and scale dependent part of the coupling constant.

In section 5.1 we review the results of the previous works and in 5.2 we compare the asymptotic expansion formulas at $a = \mu/T \ll 1$ derived in different schemes.

We give a few comments and, in particular, discuss the origin of the discrepancies between different calculations.

In Appendix we collect the relevant integrals which we encounter in the text.

2 Formal methods

2.1 Background field method

Explicit gauge invariance, which is present at the classical level in gauge field theories, is normally lost at the quantum level. This can be seen from the generating functional for QCD:

$$Z(J) = \int DAD\Psi D\bar{\Psi} D\eta D\bar{\eta} \exp\{i \int d^4x [\mathcal{L}_{QCD}(A, \Psi, \bar{\Psi}) - (G^a)^2/(2\xi) + J^{\mu a} A_{\mu a} + \bar{\eta} \partial G^a / \partial \omega^a \eta]\}, \quad (2.1)$$

where A is the gluon field, Ψ and $\bar{\Psi}$ are the fermion fields, η and $\bar{\eta}$ are ghost fields, $(G^a)^2$ is a gauge fixing term (ξ :gauge parameter), ω^a is a $SU(3)$ group parameter, and $J^{\mu a} A_{\mu a}$ is a source term. In $SU(3)$ the gauge fields transform in the following way:

$$A_\mu \rightarrow U(A_\mu - i\partial_\mu/g)U^\dagger, \quad (2.2)$$

$$\Psi \rightarrow U\Psi, \bar{\Psi} \rightarrow \bar{\Psi}U^\dagger, \quad (2.3)$$

$$\eta \rightarrow U\eta, \bar{\eta} \rightarrow \bar{\eta}U^\dagger, \quad (2.4)$$

where $U(x) = \exp[i\omega(x)^a T_a]$ is a unitary transformation, and T_a is a generator for $SU(3)$. By using transformations (2.2)-(2.4) one can see that the gauge invariance of $Z(J)$ is lost in commonly used gauges such as $\partial_\mu A_\mu = 0$.

The advantage of the background field method [20,21] is that it can maintain the explicit gauge invariance. For this purpose we divide the gluon field A_μ into a sum of a classical background field B_μ and a quantum field Q_μ

$$A_\mu = B_\mu + Q_\mu, \quad (2.5)$$

and choose for the gauge fixing term G^a the background field gauge condition

$$G^a = (D_\mu(B))^{ab} Q_b^\mu, \quad (2.6)$$

where D_μ is the covariant derivative:

$$(D_\mu)_{ab} = \partial_\mu \delta_{ab} + g f_{abc} B_\mu^c. \quad (2.7)$$

The generating functional reads

$$Z(J, B) = \int DQ D\Psi D\bar{\Psi} D\eta D\bar{\eta} \int \exp\{i \int d^4x [\mathcal{L}_{QCD}(B + Q, \Psi, \bar{\Psi}) - (G^a)^2/(2\xi) + J_\mu^a A_\mu^a + \bar{\eta} \partial G^a / \partial \omega^a \eta]\}. \quad (2.8)$$

This functional is gauge invariant, which follows from the transformations:

$$B_\mu \rightarrow U(B_\mu - i\partial_\mu/g)U^\dagger, \quad (2.9)$$

$$Q_\mu \rightarrow UQ_\mu U^\dagger, \quad (2.10)$$

$$D_\mu(B) \rightarrow UD_\mu(B)U^\dagger, \quad (2.11)$$

$$J_\mu(B) \rightarrow UJ_\mu(B)U^\dagger, \quad (2.12)$$

together with Eqs. (2.3) and (2.4). Notice that we are only changing the integration variables in Eq. (2.8).

In the background field method the quantum gauge fields and the ghost field need not be renormalized since they appear only inside loops. No vertex functions are to be considered. Thus only renormalizations

$$g_0 = Z_g g_R, \quad (2.13)$$

$$B_0 = (Z_B)^{1/2} B_R, \quad (2.14)$$

$$\xi_0 = Z_\xi \xi_R, \quad (2.15)$$

are needed.

The explicit gauge invariance of $Z(J, B)$ implies that the perturbation series is gauge invariant in every order in g_R and that the background field renormalization factor Z_B and the coupling constant renormalization Z_g are related with each other. The field tensor $F_{\mu\nu}$ in \mathcal{L}_{QCD} has to be gauge covariant and is renormalized as

$$F_{\mu\nu}^a = (Z_B)^{1/2} [\partial_\mu (B_\nu^a)_R - \partial_\nu (B_\mu^a)_R + g_R Z_g (Z_B)^{1/2} f^{abc} (B_\mu^b)_R (B_\nu^c)_R]. \quad (2.16)$$

Thus we have a relation

$$Z_g = (Z_B)^{-1/2}, \quad (2.17)$$

which enables us to calculate the evolution of coupling constant g_R in a simple way. We assume this relation to be valid also at finite temperature.

2.2 Derivation of the RG equations

From Eqs. (2.13) and (2.17) we have

$$g_0 = g_R (Z_B)^{-1/2}. \quad (2.18)$$

We use the dimensional regularization [24,23] and perform our calculations in $(4-2\varepsilon)$ dimensions. We notice from the Lagrangian of QCD that the dimension of a gluon (quark) field is $\mu^{1-\varepsilon}(\mu^{3/2-\varepsilon})$, where μ is the scale parameter. We redefine g_0 so that it becomes dimensionless and rewrite Eq. (2.18) as

$$g_0 = g_R (Z_B)^{-1/2} \mu^\varepsilon, \quad (2.19)$$

where the bare coupling g_0 does not depend on temperature T and the scale μ . A pair of RG equations results upon taking the derivative of Eq. (2.19) with respect to T and μ

$$T\partial/\partial T[g_R Z_B^{-1/2}\mu^\varepsilon] = 0, \quad (2.20)$$

$$\mu\partial/\partial\mu[g_R Z_B^{-1/2}\mu^\varepsilon] = 0. \quad (2.21)$$

These equations, which are not symmetrical in T and μ , determine the behaviour of coupling constant g_R with respect to T and μ changes. Generally Z_B has the form [22,23]

$$Z_B = 1 + \sum_{i=1}^{\infty} g_R^{2i} \left[\sum_{j=1}^i A_i^{(j)} / \varepsilon^j + f^{(i)}(\mu, T) \right], \quad (2.22)$$

where $A_j^{(j)}/\varepsilon^j$ are the divergent contributions independent on T and μ , while $f^{(j)}(\mu, T)$ are convergent temperature and scale dependent functions which vanish in the low-temperature limit.

From Eqs. (2.21) and (2.22) one can derive the following formula, by taking $\varepsilon \rightarrow 0$, (from now on we write $g = g_R$, for simplicity)

$$\mu\partial g/\partial\mu = g^3\{-A^{(1)} + (\mu/2)\partial/\partial\mu[f^{(1)}(\mu, T)]\} + O(g^5). \quad (2.23)$$

This equation is reduced to the ordinary one-loop RG equation in the zero-temperature limit since in this limit the function $f^{(1)}(\mu, T=0)$ vanishes.

Similarly one can derive the RG equation for T from Eqs. (2.20) and (2.22)

$$T\partial g/\partial T = (g^3/2)T\partial/\partial T[f^{(1)}(\mu, T)] + O(g^5). \quad (2.24)$$

Equations (2.23) and (2.24) constitute the coupled RG equations.

2.3 Solution of the coupled RG equations

The solution to Eqs. (2.23) and (2.24) can be found in a straightforward way by integrating Eq. (2.23) from μ_0 to μ and Eq. (2.24) from T_0 to T . We have

$$\begin{aligned} 1/[2g^2(\mu_0, T)] - 1/[2g^2(\mu, T)] &= -A^{(1)} \ln(\mu/\mu_0) \\ &+ [f^{(1)}(\mu, T) - f^{(1)}(\mu_0, T)]/2, \end{aligned} \quad (2.25)$$

$$1/[2g^2(\mu, T_0)] - 1/[2g^2(\mu, T)] = [f^{(1)}(\mu, T) - f^{(1)}(\mu, T_0)]/2. \quad (2.26)$$

Putting T equal to T_0 in Eq. (2.25), and using Eq. (2.26) we arrive at the desired solution

$$\begin{aligned} [g^2(\mu, T)]^{-1} &= [g^2(\mu_0, T_0)]^{-1} + 2A^{(1)} \ln(\mu/\mu_0) \\ &- [f^{(1)}(\mu, T) - f^{(1)}(\mu_0, T_0)]. \end{aligned} \quad (2.27)$$

This equation describes the evolution of the QCD coupling constant as a function of the momentum scale and the arbitrary temperature [5-9]. (μ_0, T_0) denotes the reference point.

2.4 Imaginary time formalism

The Feynman rules at finite temperature $T(= 1/\beta)$ are derived from the generating functional

$$Z(J) = c \int DQ D\Psi D\bar{\Psi} D\eta D\bar{\eta} \exp\left[\int_0^\beta d\tau \int d^3x \{\mathcal{L}(x, \tau) - [G^a(x, \tau)]^2/(2\xi) + \bar{\eta}(x, \tau) \partial G^a / \partial \omega^a \eta(x, \tau)\}\right], \quad (2.28)$$

where $\eta, \bar{\eta}, Q$ are periodic fields, while $\Psi, \bar{\Psi}$ are antiperiodic fields.

This functional differs from Eq. (2.8) only in that time τ is now imaginary *it*. Infinite time domain has been compactified to the finite interval $[0, \beta]$. From Eq. (2.28) one can formulate the Feynman rules at finite temperature by modifying the ones at zero temperature in the following way [25]: For the loop integral we have

$$\int d^4p/(2\pi)^4 \rightarrow (i/\beta) \sum_n \int d^3p/(2\pi)^3, \quad (2.29)$$

and for the time-component of 4-momentum we have for bosons and ghosts:

$$p_0 = i2n\pi/\beta, \quad (2.30)$$

and for fermions:

$$p_0 = i(2n+1)\pi/\beta + \mu_{ch}, \quad (2.31)$$

where μ_{ch} is a chemical potential. The frequency sums for bosons and fermions in Eq. (2.29) are readily changed to contour integrals [26]: We have for bosons and ghosts

$$\begin{aligned} (i/\beta) \sum_{n=-\infty}^{\infty} f(p_0 = i2n\pi/\beta) &= (1/2\pi) \int_{-i\infty+\varepsilon}^{i\infty+\varepsilon} dp_0 f(p_0) N_B(p_0) \\ &- (1/2\pi) \int_{-i\infty-\varepsilon}^{i\infty-\varepsilon} dp_0 f(p_0) N_B(-p_0) + (1/2\pi) \int_{-i\infty}^{i\infty} dp_0 f(p_0), \end{aligned} \quad (2.32)$$

where

$$N_B(p_0) = 1/[\exp(\beta p_0) - 1], \quad (2.33)$$

and for fermions

$$\begin{aligned} (i/\beta) \sum_{n=-\infty}^{\infty} f[p_0 = i(2n+1)\pi/\beta + \mu_{ch}] &= -(1/2\pi) \\ &\times \int_{-i\infty+\varepsilon}^{i\infty+\varepsilon} dp_0 f(p_0 + \mu_{ch}) N_F(p_0) + (1/2\pi) \int_{-i\infty-\varepsilon}^{i\infty-\varepsilon} dp_0 \\ &\times f(p_0 + \mu_{ch}) N_F(-p_0) + (1/2\pi) \int_{-i\infty}^{i\infty} dp_0 f(p_0), \end{aligned} \quad (2.34)$$

where

$$N_F(p_0) = 1/[\exp(\beta p_0) + 1]. \quad (2.35)$$

The first two terms in the right-hand side of Eq. (2.34) correspond to particle and anti-particle contributions respectively and vanish at $T = 0$. Since we will be only interested in phenomena in an environment where the sum of quantum numbers is not conserved, hereafter we put μ_{ch} equals to zero. Also we have omitted the term which gives the finite density contribution at $T = 0$.

3 Polarization tensor

3.1 Structure of the polarization tensor

At a zero temperature environment the polarization tensor is Lorentz invariant and can be expressed as [22]

$$\Pi_{\mu\nu}^{ab} = \delta^{ab}\Pi_{\mu\nu}, \quad (3.1)$$

where

$$\Pi_{\mu\nu} = \Pi(k^2 g_{\mu\nu} - k_\mu k_\nu). \quad (3.2)$$

The zero temperature polarization tensor is transverse with respect to k (current conservation):

$$k_\mu \Pi_{\mu\nu} = 0. \quad (3.3)$$

At finite temperature, in the presence of matter, the Lorentz invariance is lost and the polarization tensor can only be $O(3)$ rotational invariant [27,17]. Then the polarization tensor can generally depend only on 4 independent quantities, which we can choose, for example, Π_{00} , $k_i \Pi_{0i}$ and the two scalars Π_L and Π_T appearing in

$$\Pi_{ij} = \Pi_T(\delta_{ij} - k_i k_j / \mathbf{k}^2) + \Pi_L k_i k_j / \mathbf{k}^2. \quad (3.4)$$

At finite temperature whether the transversality condition is satisfied or not depends on the gauge used. The polarization tensor is not transversal, e.g., in the Coulomb gauge ($\partial_i A_i = 0$), but is transversal at the one-loop level in the temporal axial gauge ($A_0 = 0$), and in every order of the perturbative calculations in the background field gauge [17]. The transversality condition (3.3) restricts the structure of the polarization tensor. From Eq. (3.3) we have

$$k_0 \Pi_{00} = k_i \Pi_{i0}, \quad (3.5)$$

and

$$k_0^2 \Pi_{00} = k_i k_j \Pi_{ij}. \quad (3.6)$$

Using Eqs. (3.4) and (3.6) we obtain

$$\Pi_L = k_0^2 \Pi_{00} / \mathbf{k}^2. \quad (3.7)$$

For the coefficient of the transversal part of the polarization tensor we get from Eqs. (3.4) and (3.7) an expression

$$\Pi_T = (\Pi_{ii} - k_0^2 \Pi_{00}/\mathbf{k}^2)/2. \quad (3.8)$$

By using $\Pi_{ii} = (\Pi_{\mu\mu} - \Pi_{00})$, we rewrite Eq. (3.8) as

$$\Pi_T = [\Pi_{\mu\mu} - \Pi_{00}(1 + k_0^2/\mathbf{k}^2)]/2. \quad (3.9)$$

Thus in Eqs. (3.4)-(3.9) we have derived the general form of the temperature dependent $O(3)$ symmetric polarization tensor in the background field method.

The polarization tensor can be splitted into a sum of a temperature independent (vacuum) part and a temperature and scale dependent (matter) part [4]:

$$\Pi_{\mu\nu} = \Pi_{\mu\nu}|_{vac} + \Pi_{\mu\nu}|_{matt}. \quad (3.10)$$

At zero temperature limit the temperature dependent matter part vanishes.

The structure of the polarization tensor at zero temperature was given in Eqs. (3.1) and (3.2). The polarization tensor is related to the gluon propagator, $D_{\mu\nu}$, as

$$\Pi_{\mu\nu} = D_{R\mu\nu}^{-1} - D_{0\mu\nu}^{-1} = \Pi D_{R\mu\nu}^{-1}. \quad (3.11)$$

The relation between the bare and the renormalized propagators (see Eq. (2.14)) is

$$D_{0\mu\nu} = Z_B^{-1} D_{R\mu\nu}, \quad (3.12)$$

which together with Eq. (3.11) leads us to

$$Z_B = 1 - \Pi. \quad (3.13)$$

Thus the renormalization constant Z_B can be obtained from the (background field) gluon self-energy tensor.

In investigating the behaviour of the coupling constant one has to include the temperature dependent parts of the polarization tensor in the renormalization constant Z_B via Eq. (3.13). Although the finite renormalization is somehow arbitrary one has to follow some rule or prescription in every order of perturbation to be consistent [23].

At finite temperature we encounter another type of ambiguity which is caused by the lack of the Lorentz invariance. In order to define the polarization tensor at finite temperature we generalize Eq.(3.13) for $T = 0$ as:

$$Z_B = 1 - \Pi|_{vac} - \Pi|_{matt}, \quad (3.14)$$

where we have either

$$\Pi|_{matt} = \Pi_{00}/\mathbf{k}^2, \quad (3.15)$$

or

$$\Pi|_{matt} = \Pi_T/\mathbf{k}^2. \quad (3.16)$$

Naturally at $T = 0$, Eq.(3.14) reduces to Eq.(3.13). Π_{00} and Π_T are not connected with each other and in general there is no a priori way to decide which one is more natural.

3.2 The polarization tensor at the one-loop level

Our working formulas, which allow to analyze the evolution of QCD coupling constant at finite temperature, are: Eq.(2.22), Eq.(2.27) and Eqs. (3.14)-(3.16) for $T \neq 0$ case [Eq.(3.13) for $T = 0$ case]. Accordingly we have to evaluate the self-energy diagrams in Fig.1 to get the renormalization factor for Z_B up to the one-loop order.

The Feynman rules for the interaction vertices are the same as in the zero temperature case, and therefore identical to those given in [21]. Evaluating the one-loop diagrams (1a)-(1d) in the Feynman gauge (i.e., we set $\xi = 1$ in the gluon propagator) we find for the boson contributions in the polarization tensor

$$\begin{aligned} \Pi_{\mu\nu}|_{boson} = & ig^2 N \int d^4 p / (2\pi)^4 [4g_{\mu\nu} k^2 + 2(k_\mu p_\nu + k_\nu p_\mu) \\ & + 4p_\mu p_\nu - 3k_\mu k_\nu - 2(k+p)^2 g_{\mu\nu}] / D, \end{aligned} \quad (3.17)$$

where

$$D = (k+p)^2 p^2. \quad (3.18)$$

For the fermion loop, neglecting the quark masses compared to momentum scale and temperature, we have from the diagram (1e):

$$\begin{aligned} \Pi_{\mu\nu}|_{fermion} = & -4ig^2 T_F n_F \int d^4 p / (2\pi)^4 \{k_\mu p_\nu \\ & + k_\nu p_\mu + 2p_\mu p_\nu - g_{\mu\nu}[(kp) + p^2]\} / D, \end{aligned} \quad (3.19)$$

where $T_F = 1/2$ for $SU(3)$. We have also included n_F (number of flavours) in Eq. (3.19).

Our polarization tensor satisfies the transversality condition

$$k_\mu \Pi_{\mu\nu}|_{boson} = k_\mu \Pi_{\mu\nu}|_{fermion} = 0. \quad (3.20)$$

This can be shown explicitly as follows: From Eq. (3.17) we have

$$\begin{aligned} \Pi_{\mu\nu}|_{boson} = & ig^2 N \int d^4 p / (2\pi)^4 [-k_\nu / p^2 \\ & - (k+p)_\nu / (k+p)^2 + 2p_\nu / p^2 - p_\nu / (k+p)^2]. \end{aligned} \quad (3.21)$$

Replacing $k+p \rightarrow p$ in the second term we have

$$\Pi_{\mu\nu}|_{boson} = -ig^2 N \int d^4 p / (2\pi)^4 [(k-p)_\nu / p^2 + p_\nu / (k+p)^2]. \quad (3.22)$$

Then by changing $k-p \rightarrow -p$ in the first term, we immediately come to Eq. (3.20) for the boson contributions. Similarly one can prove the transversality condition for the fermion polarization tensor. In the next subsection we will extract from $\Pi_{\mu\nu}$ the information on the vacuum part of the polarization tensor.

3.3 Vacuum part of the polarization tensor

Using a standard technique and introducing the Feynman parametrization in Eqs. (3.17) and (3.19), namely

$$1/(ab) = \int_0^1 dx \{1/[ax + b(1-x)]^2\}, \quad (3.23)$$

we have

$$1/D = 1/[p^2(k+p)^2] = \int_0^1 dx \{1/[(p+kx)^2 + k^2x(1-x)]^2\}. \quad (3.24)$$

Changing variable $p \rightarrow p-kx$ we get the following formula for the boson contributions from Eq. (3.17):

$$\begin{aligned} \Pi_{\mu\nu}|_{boson} = & ig^2 N \int d^4p / (2\pi)^4 \int_0^1 dx [4g_{\mu\nu}k^2 + 2(1-2x)(k_\mu p_\nu + k_\nu p_\mu) \\ & + 4p_\mu p_\nu + (-3-4x+4x^2)k_\mu k_\nu] / (p^2 + K^2)^2, \end{aligned} \quad (3.25)$$

where

$$K^2 = k^2x(1-x). \quad (3.26)$$

Similarly we get the formula for the fermion contributions from Eq. (3.19):

$$\begin{aligned} \Pi_{\mu\nu}|_{fermion} = & -2ig^2 n_F \int d^4p / (2\pi)^4 \int_0^1 dx [2(-x+x^2)k_\mu k_\nu + (1-2x) \\ & \times (k_\mu p_\nu + k_\nu p_\mu) - g_{\mu\nu}(kp) + 2p_\mu p_\nu + g_{\mu\nu}k^2x] / (p^2 + K^2)^2. \end{aligned} \quad (3.27)$$

In these equations for the vacuum parts we have dropped the terms proportional to $1/p^2$ and $1/(k+p)^2$, which turn out to be zero after simple calculation [see Eq. (A.1) in the Appendix]. Notice that all of the integrals are ultraviolet divergent and thus have to be regularized. For this purpose we employ the dimensional regularization [24,23], which preserves gauge symmetries explicitly. The integrals for the vacuum parts become Euclidean if we change $ip_0 \rightarrow p_4$ and thus can be easily evaluated. We obtain the following results:

$$\Pi_{\mu\nu}|_{boson} = -11g^2 N / (3\alpha)(g_{\mu\nu}k^2 - k_\mu k_\nu) / \varepsilon + O(1), \quad (3.28)$$

and

$$\Pi_{\mu\nu}|_{fermion} = 2g^2 n_F / (3\alpha)(g_{\mu\nu}k^2 - k_\mu k_\nu) / \varepsilon + O(1), \quad (3.29)$$

where

$$\alpha = (4\pi)^2. \quad (3.30)$$

Combining Eqs. (3.28) and (3.29) and taking into account Eqs.(2.22), (3.2) and (3.13), we get for $A^{(1)}$ in Eq. (2.27) as:

$$A^{(1)} = (11N - 2n_F) / (3\alpha), \quad (3.31)$$

which is, as expected, in accord with Eq. (1.2) for zero temperature. In concluding this subsection let us emphasize the simplicity of the calculation by the background field method in contrast to the conventional methods, e.g., in the covariant gauge [7-9].

4 Temperature dependent parts of the polarization tensor

4.1 Calculation of Π_{00} and $\Pi_{\mu\mu}$

In this subsection we calculate Π_{00} and $\Pi_{\mu\mu}$, without using the Feynman parametrization. To specify the subtraction point we employ the static limit of zero external energy, which is commonly used in the literature [5]. In this prescription the momentum k is specified to be space-like $k = (0, k_i) (i = 1, 2, 3)$ with $k^2 = -\mu^2$. Such a choice enables us to determine the static properties. Using Eqs. (3.17) and (3.19) and the integrals from the Appendix we obtain for Π_{00}

$$\begin{aligned}\Pi_{00} &= 2ig^2N \int d^4p/(2\pi)^4 [2k^2 + 2p_0^2 - (p+k)^2]/D|_{boson} \\ &+ 2ig^2n_F \int d^4p/(2\pi)^4 [-2p_0^2 + (kp) + p^2]/D|_{fermion} \\ &= g^2T^2(N/6 + n_F/12) - 2g^2N\mu^2[4F_{B0}(a) - F_{B2}(a)] \\ &- 2g^2n_F\mu^2[F_{F0}(a) - F_{F2}(a)],\end{aligned}\tag{4.1}$$

with $a = \beta\mu$. Similarly we derive an expression for $\Pi_{\mu\mu}$:

$$\begin{aligned}\Pi_{\mu\mu} &= ig^2N \int d^4p/(2\pi)^4 [5k^2 - 12(kp) - 4p^2]/D|_{boson} \\ &+ 4ig^2n_F \int d^4p/(2\pi)^4 [(kp) + p^2]/D|_{fermion} \\ &= g^2T^2(N/3 + n_F/6) - 2g^2\mu^2[11NF_{B0}(a) + 2n_FF_{F0}(a)],\end{aligned}\tag{4.2}$$

where we define the boson functions

$$F_{Bn}(\beta\mu) = (1/\alpha) \int_0^\infty dx x^n N_B(\mu x/2)L,\tag{4.3}$$

with

$$L = \ln |(1+x)/(1-x)|.\tag{4.4}$$

The integrals (4.3) are not ultraviolet divergent and hence do not give infinite $1/\varepsilon$ contribution. The fermion functions $F_{Fn}(a)$ are defined by replacing in Eq. (4.3)

$$N_B(\mu x/2) \rightarrow N_F(\mu x/2).\tag{4.5}$$

4.2 Temperature dependent part of the coupling constant

As pointed out in section 3.1 we encounter an ambiguity in determining the renormalization constant Z_B as a direct consequence of the lack of Lorentz invariance. Here we write two formulas, one derived from Π_{00} and the other one from Π_T . From Eq. (3.15) we have ($a = \beta\mu$)

$$\begin{aligned} -f^{(1)}(\mu, T) &= (N/6 + n_F/12)/a^2 - 2N[4F_{B0}(a) - F_{B2}(a)] \\ &\quad - 2n_F[F_{F0}(a) - F_{F2}(a)], \end{aligned} \quad (4.6)$$

and the other one from Eq.(3.16) and $\Pi_T = (\Pi_{\mu\mu} - \Pi_{00})/2$

$$\begin{aligned} -f^{(1)}(\mu, T) &= (N/12 + n_F/24)/a^2 - N[7F_{B0}(a) + F_{B2}(a)] \\ &\quad - n_F[F_{F0}(a) + F_{F2}(a)]. \end{aligned} \quad (4.7)$$

The result for Eq. (4.7) was reported in a short communication in [18](where an overall factor of $1/2$ was missing in the right-hand side of formulas (8) and (9)).

5 Discussion

5.1 Review on the results of previous works

Gendenshtein [5] calculated the QCD coupling constant at finite temperature in the one-loop approximation by using the RG equation, the dimensional regularization, and the covariant gauge with a space-like normalization momentum $p^\mu = (0, \mu)$ and obtained

$$A^{(1)} = (11N - 2n_F)/(3\alpha), \quad (5.1)$$

as in Eq. (3.31), and

$$-f^{(1)}(\mu, T) = (N/3 + n_F/6)/a^2, \quad (5.2)$$

where we have presented the results using our notations.

Kajantie et al. [6] studied the gauge field part of QCD with N colors. They used the $A_0 = 0$ gauge and defined two renormalization schemes by writing

$$D_{\mu\nu}^{-1} = D_{0\mu\nu}^{-1}(Z_A - G/p^2) - (F - G)P_{\mu\nu}^L, \quad (5.3)$$

where $D_{\mu\nu}$ is the gluon propagator and F, G and $P_{\mu\nu}^L$ come from the polarization tensor

$$\Pi_{\mu\nu} = FP_{\mu\nu}^L + GP_{\mu\nu}^T. \quad (5.4)$$

In the “magnetic prescription” they fixed the propagator at the point $p^\mu = (0, \mu)$ and had for the temperature depending function the expansions (without quarks)

$$-f^{(1)}(\mu, T) = 5N/(16a), \text{ for } a \ll 1, \quad (5.5)$$

$$-f^{(1)}(\mu, T) = N[17/(90a^2) + 83\alpha/(6300a^4)], \text{ for } a \gg 1. \quad (5.6)$$

In the so-called “electric prescription” they derived the coupling constant from F as

$$-f^{(1)}(\mu, T) = N[1/(3a^2) - 1/(4a) - 22(\ln a)/(3\alpha)], \text{ for } a \ll 1 \quad (5.7)$$

$$-f^{(1)}(\mu, T) = -N[1/(18a^2) - 11\alpha/(900a^4)], \text{ for } a \gg 1. \quad (5.8)$$

Notice here the change of sign in the high T behaviour. They concluded that the “magnetic prescription” is more natural than the “electric” one because the former uses the physical part of the gluon propagator. They also argued that one has to choose $\mu \cong 3T$ in the thermal equilibrium. In that case the effect of the temperature dependent parts becomes negligible.

Nakagawa, Niégawa and Yokota [7] used the real-time formalism and studied the scale-parameter ratios $\Lambda(a)/\Lambda$ derived from different vertices in 4-flavour QCD. They used the covariant gauge and found that the ratios derived from the three-gluon vertex and the gluon-ghost vertex show just the opposite behaviour than the one derived from the gluon-quark vertex. Namely in the former case one gets a growing ratio while in the latter case a decreasing ratio.

Fujimoto and Yamada [8] used the real-time formalism and derived the temperature depending coupling constant from the gauge-independent Wilson loop. At $a \ll 1$, it reads as (without quark contributions)

$$-f^{(1)}(\mu, T) = C[1/(3a^2) - 1/(4a) - 22(\ln a)/(3\alpha)]. \quad (5.9)$$

The same authors have discussed the finite temperature RG equations [9] in the one-loop approximation, using the real-time formalism, the covariant gauge and the dimensional regularization. From the gluon propagator and three-gluon vertex they obtained:

$$\begin{aligned} -f^{(1)}(\mu, T) = & (C + T_f)/(4a^2) + C[-23/3F_{B0}(a) - 3F_{B2}(a) - 14/3G_{B0}(a) \\ & + 32G_{B2}(a)] + 2T_f[F_{F0}(a) - 3F_{F2}(a) + 152/9G_{F0}(a)], \end{aligned} \quad (5.10)$$

where $T_f = 1/2$, and $C = N$ for $SU(N)$. The coefficient in front of $F_{B0}(a)$ was originally $4/3$. The error was pointed out in Ref. [10].

From the fermion propagator and fermion-gluon vertex:

$$\begin{aligned} -f^{(1)}(\mu, T) = & (C + 3C_f + 2T_f)/(12a^2) - 2(C_f + 11C)F_{B0}(a)/3 \\ & - 16(C_f + 10C)G_{B0}(a)/9 - 32CG_{B2}(a) + 2(-5C/6 + C_f/3 + T_f) \\ & \times F_{F0}(a) - 4(8C_f + 17C)G_{F0}(a)/9 + 16CG_{F2}(a), \end{aligned} \quad (5.11)$$

where $C_f = (N^2 - 1)/(2N)$ for $SU(N)$.

From ghost propagator and ghost-gluon vertex:

$$\begin{aligned} -f^{(1)}(\mu, T) = & (C + T_f)/(12a^2) - C[7F_{B0}(a) + F_{B2}(a) \\ & + 2G_{B0}(a)] - 2T_f[F_{F0}(a) + F_{F2}(a)]. \end{aligned} \quad (5.12)$$

In the above equations the boson functions $F_{Bn}(a)$ are defined in Eq.(4.3), while $G_{Bn}(a)$ are defined as

$$G_{Bn}(a) = (2/\alpha) \int_0^\infty dx \int_0^1 dy x^{n+1} N_B(\mu x/2) / [x^2(y^2 + 3) - 1]. \quad (5.13)$$

Fermion functions $G_{Fn}(a)$ are defined by replacing $N_B(\mu x/2)$ by $N_F(\mu x/2)$ in Eq. (5.13) as $F_{Bn}(a)$ was defined from Eq. (4.3). The functions $G_{in}(A)$ ($i = B, F$) do not appear in our results Eqs. (4.6) and (4.7), since they come from trigluon renormalization only. [Note an amusing coincidence: $-f^{(1)}$ in Eq. (4.6) = $-f^{(1)}[G_{B0}(a) \rightarrow 0]$ of Eq. (5.12), both of which are derived from very different prescriptions.]

Stephens et al. [19] performed a background field one-loop calculation of gauge invariant beta functions at finite temperature, using the retarded/advanced formalism developed by Aurenche and Becherrawy [28]. In terms of our notations, their result reads as:

$$\begin{aligned} -f^{(1)}(\mu, T) = & (N/12 + n_F/24)/a^2 - N[21/4F_{B0}(a) \\ & + F_{B2}(a) + 7/2G_{B1}(a)] - n_F[F_{F0}(a) + F_{F2}(a)]. \end{aligned} \quad (5.14)$$

The numerical coefficients of the leading terms of the high-temperature expansion and the fermion parts are in complete agreement with our result in Eq.(4.7). In the boson parts, however, there are some numerical discrepancies with our result.

5.2 Comparison of asymptotic expansions

In order to compare our results with those mentioned in section 5.1 we derive the asymptotic expansions for $-f^{(1)}(\mu, T)$. The asymptotic expansions for Eqs. (5.10)-(5.12) at $a = \mu/T \ll 1$ in the high-temperature regime read as follows:

$$\begin{aligned} -f^{(1)}(\mu, T) = & \pi C/(9\sqrt{3}a^2) - 23C/(48a) + [(-22 \\ & + \pi/3\sqrt{3})C + (18 - 38\pi/9\sqrt{3})T_f](\ln a)/(3\alpha), \end{aligned} \quad (5.15)$$

$$\begin{aligned} -f^{(1)}(\mu, T) = & [(1 - \pi/\sqrt{3})C + 3C_f]/(12a^2) - (11C + C_f)/(24a) \\ & - [(23 + 82\pi/9\sqrt{3})C + (4 - 8\pi/9\sqrt{3})C_f + 4T_f](\ln a)/(3\alpha), \end{aligned} \quad (5.16)$$

$$-f^{(1)}(\mu, T) = -7C/(16a) - [C(22 + \pi/\sqrt{3}) - 8T_f](\ln a)/(3\alpha). \quad (5.17)$$

Next we derive the asymptotic expansions for our results, i.e., Eqs. (4.6) and (4.7) at $a \ll 1$, by using the following high temperature expansions [9]:

$$F_{B0}(a) = 1/(16a) + (\ell_1 - 1)/\alpha, \quad (5.18)$$

$$F_{B2}(a) = 1/(12a^2) + (\ell_1 - 1/6)/(3\alpha), \quad (5.19)$$

$$F_{F0}(a) = -(\ell_2 - 1/2)/\alpha, \quad (5.20)$$

$$F_{F2}(a) = 1/(24a^2) - (\ell_2 - 1/6)/(3\alpha), \quad (5.21)$$

where

$$\ell_1 = \ln(a/4\pi) + \gamma, \quad (5.22)$$

$$\ell_2 = \ln(a/\pi) + \gamma. \quad (5.23)$$

We find the asymptotic formula from Eq. (4.6)

$$\begin{aligned} -f^{(1)}(\mu, T) &= (N/3 + n_F/6)/a^2 - N/(2a) \\ &- [N(22\ell_1 - 71/3) - 4n_F(\ell_2 - 2/3)]/(3\alpha). \end{aligned} \quad (5.24)$$

The T^2 term coincides with the one of Gendenshtein [5], i.e., Eq. (5.1). The gluon part in this asymptotic formula is consistent with the result of Elze et al. [17] derived in the background field method, which in our notations reads

$$-f^{(1)}(\mu, T) = N[1/(3a^2) - 1/(2a) - 22(\ln a)/(3\alpha) + \dots], \text{ for } a \ll 1. \quad (5.25)$$

This coincides with the result of Nadkharni [12], who has the terms up to $O(1/a)$. The delicate reason why in the background field method we have a factor -1/2 in front of the second term proportional to $1/a$, while in some results [see Eqs. (5.7) and (5.9)] it is equal to -1/4 is clarified by Elze et al. in [17].

Next from Eq. (4.7) we find the asymptotic formula:

$$-f^{(1)}(\mu, T) = -7N/(16a) - [N(22\ell_1 - 127/6) - 4n_F(\ell_2 - 5/12)]/(3\alpha). \quad (5.26)$$

The behaviour of the transverse polarization tensor Π_T in the infrared region is known to have a form [17]

$$\lim \Pi_T(0, \mathbf{k})/\mu^2 = g^2[-cN/a + O(\ln a)]. \quad (5.27)$$

The factor c has been calculated in different gauges. In the covariant ξ -gauge its value is [27]

$$c = (9 + 2\xi + \xi^2)/64, \quad (5.28)$$

whereas in the temporal axial gauge it is [6]

$$c = 5/16. \quad (5.29)$$

Our formula (5.26) at small a behaves as Eq. (5.27) with

$$c = -7/16, \text{ for bosons,} \quad (5.30)$$

and

$$c = 0, \text{ for fermions.} \quad (5.31)$$

These results can be read also from the results of Refs. [9] [see Eq. (5.17)] and [12]. The relation (5.31) was also noticed by Elze et al. in [17]. We remark here that our c is negative and hence no spurious pole appears in the transversal propagator

$$D_{Tij}(0, \mathbf{k}) = -(\delta_{ij} - k_i k_j / \mathbf{k}^2) / [\mathbf{k}^2 - \Pi_T(0, \mathbf{k})], \quad (5.32)$$

in contrast to the covariant and the temporal axial gauge cases.

It is a known fact that the T^2 terms are gauge independent and the gauge parameter dependence starts at $\sim T$ order [13]. Thus the fact that the T^2 terms in Eqs. (5.15) and (5.16) are different from others, e.g. Eq. (5.1) should not originate from the gauge choices.

5.3 Concluding remarks

In concluding the paper we give a few comments:

1) In sections 5.1 and 5.2 we have seen that the temperature dependence of the QCD coupling constant is very sensitive to the prescription chosen. This is not a trivial issue, because all the results obtained hitherto also heavily depends on the vertex chosen (i.e., the trigluon, the ghost-gluon, or the quark-gluon vertex) to satisfy the renormalization condition of the QCD coupling constant. Furthermore in some gauges, e.g., in the Coulomb gauge, the transversality condition (3.3) on the polarization tensor does not hold and hence the structure of the polarization tensor in such gauges is different from that which satisfies the condition.

One of the reasons why we encounter different results in the literature is that the broken Lorentz invariance has not been treated as we have done in Eqs. (3.15) and (3.16). For example in deriving Eqs. (5.10)-(5.12) the authors of Ref.[9] have extracted the gluon and the vertex renormalization constants in front of Lorentz invariant structures not paying attention to the break down of Lorentz invariance.

Another possible source for discrepancies is that the Ward identities are used at finite temperature for different renormalization constants. Remember that the derivation of the Ward identity is based on the gauge invariance and also on the Lorentz invariance at $T = 0$. Thus it is not a surprise that results from different gauges or even (within the same gauge) from different vertices are totally different.

The correspondence between the imaginary and real time formalisms has been investigated in detail [14-16]. The choice of the formalism can also be the source of the discrepancies under study was pointed out in these references.

To clarify the issue of choosing a suitable renormalization prescription, one would need to compute the two-loop contributions to the coupling constant at finite temperature. It was shown, in particular, in the massive $O(N)$ scalar model that the one-loop result is drastically changed by two-loop contributions at high T and in zero momentum limit [29].

2) As we have seen the function $f^{(1)}(\mu, T)$ shows a power-like T -dependence instead of a logarithmic fall-off as a function of μ . Thus we have

$$\begin{aligned} & -[f^{(1)}(\mu, T) - f^{(1)}(\mu_0, T_0)] \\ &= \sum_{n=1,2} c_n [(T/\mu)^n - (T_0/\mu_0)^n] + \dots \end{aligned} \quad (5.33)$$

However, if a relation $T = (const)\mu$ holds, where $(const) \simeq 1/3$ [6], then one would have a logarithmic T -dependence like in a zero temperature environment. Such a relation holds if one is dealing with the thermal equilibrium. In a thermostat one cannot speak about individuals and thus μ is not a measurable quantity.

We have derived Eq. (2.27) using two essentially different RG equations (2.23) and (2.24). Suppose we had used only one RG equation for T , i.e., Eq. (2.24), then we would have instead of Eq. (5.33)

$$\begin{aligned} & -[f^{(1)}(\mu, T) - f^{(1)}(\mu_0, T)] \\ &= \sum_{n=1,2} c_n [(T/\mu)^n - (T/\mu_0)^n] + \dots \end{aligned} \quad (5.34)$$

In this case we would have a power-like dependence even with the relation $T = (const)\mu$.

3) Since in a thermostat T is a measurable quantity, though μ is not as, stated in 2), we should be able to define the coupling constant as a function of only T . This could be done by calculating an expectation value of the coupling constant. In that case as a probability density function we could use either the Bose or the Fermi distribution. However such an expectation value of the coupling constant could not be used to a scattering process in a thermostat wherein a particle with some definite energy enters.

We would like also to mention that all the machinery for the evolution of the running coupling constant at finite temperature can be analogously applied for the case of a quantum field theory (such as QCD) at finite energy as formulated in [30].

In conclusion, we note in accordance with the previous observations that there is in fact no unique way to define a temperature depending QCD coupling constant and the issue of finding its reasonable prescription is left as a subject of further investigation.

Appendix

The formulas for the integrals which we encounter in the text are assembled in this Appendix.

$$\int d^4p/(2\pi)^4[1/p^2]|_{vac} = 0, \quad (A.1)$$

$$\int d^4p/(2\pi)^4[1/p^2]|_{boson} = iT^2/12, \quad (A.2)$$

$$\int d^4p/(2\pi)^4[1/p^2]|_{fermion} = -iT^2/24, \quad (A.3)$$

with $\mu_{ch} = 0$.

In the following we use the static space-like prescription specified by $k_0 = 0$ and $k^2 = -\mu^2$:

$$\int d^4p/(2\pi)^4(1/D)|_{boson} = -2iF_{B0}(a), \quad (A.4)$$

where $F_{B0}(a)$ is defined in Eq. (4.3).

$$\int d^4p/(2\pi)^4[(kp)/D]|_{boson} = -i\mu^2F_{B0}(a), \quad (A.5)$$

$$\int d^4p/(2\pi)^4(p_0^2/D)|_{boson} = -i(\mu^2/2)F_{B0}(a). \quad (A.6)$$

For the fermions we have similar results as in Eqs. (A.4), (A.5) and (A.6) in which we just replace $N_B(\mu x/2)$ by $N_F(\mu x/2)$ and change the sign in front of the equations.

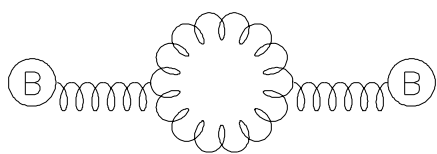
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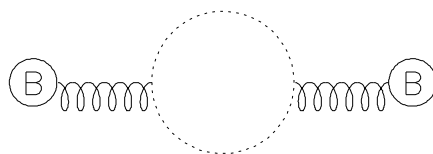
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Figure Caption

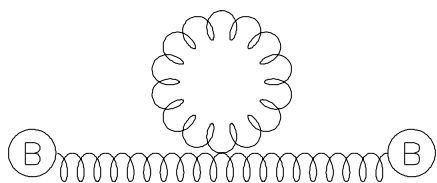
Fig.1. Diagrams for the one-loop calculation of the background field renormalization factor Z_B : a) Gluon loop; b) Ghost loop; c) Gluon loop from the 4-gluon vertex; d) Ghost loop from the 2-gluon 2-ghost vertex; e) Fermion loop. Wavy lines are quantum gauge field propagators. Wavy lines terminating in a "B" represent external gauge particles. Solid lines are fermion propagators and dashed lines represent ghost propagators.



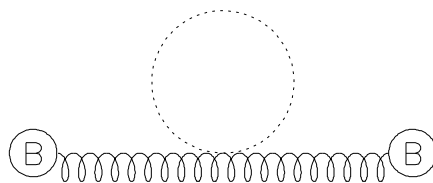
a)



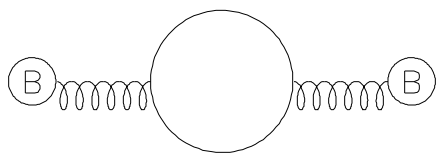
b)



c)



d)



e)

Fig. 1